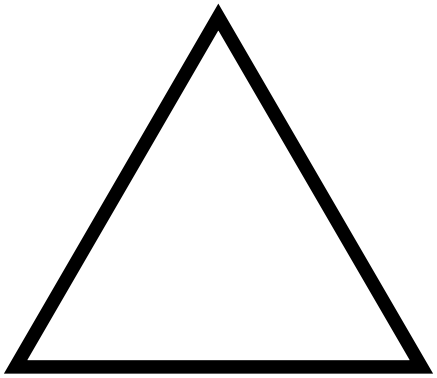
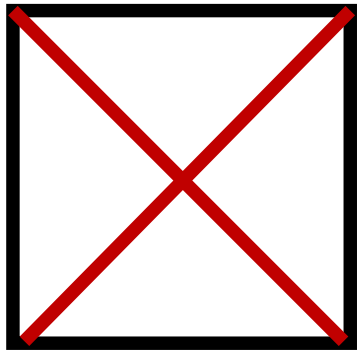
# Emory Math Circle

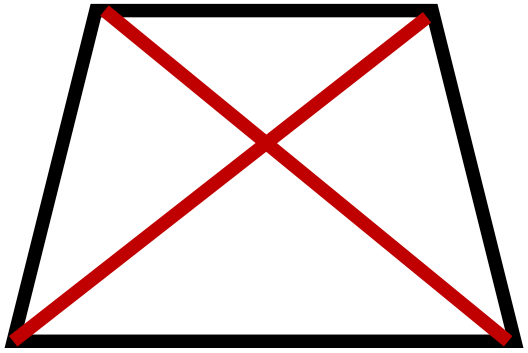
What is your favourite thing about Spring?

# Warmup

- A diagonal for a polygon is a segment connecting two non-adjacent vertices
- Example: A triangle has NO diagonals
- Example: A quadrilateral has 2 diagonals



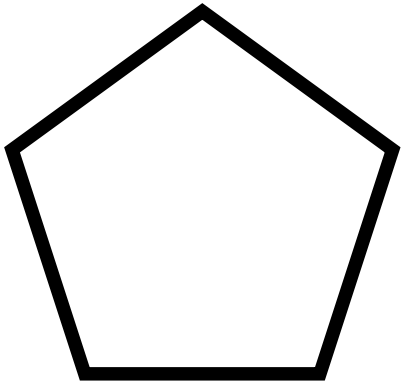


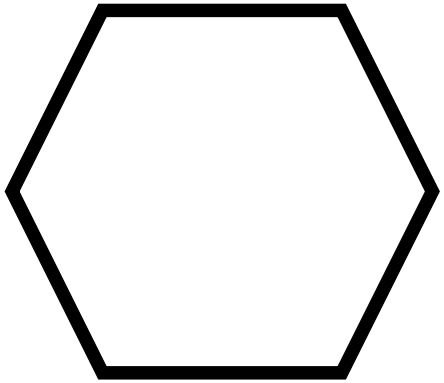


# Warmup II

- How many diagonals in a pentagon (5 sides)?
- How many in a hexagon (6 sides)?

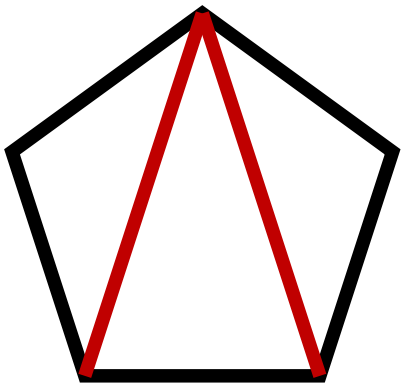
- How many in an $n$-gon ($n$ sides)?

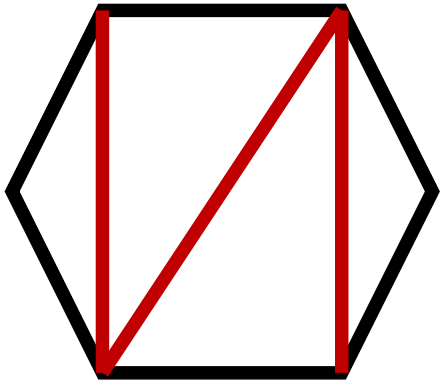




# Triangulation

- We can split any polygon into triangles by drawing diagonals
- This is called triangulation!



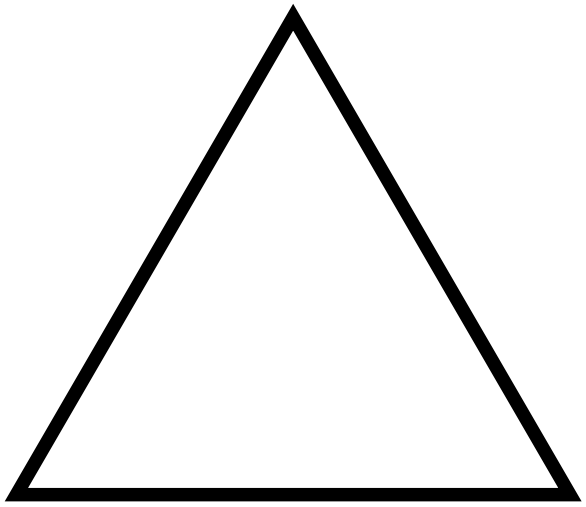


# Breakout Rooms

We will first look at how many triangulations a polygon has.

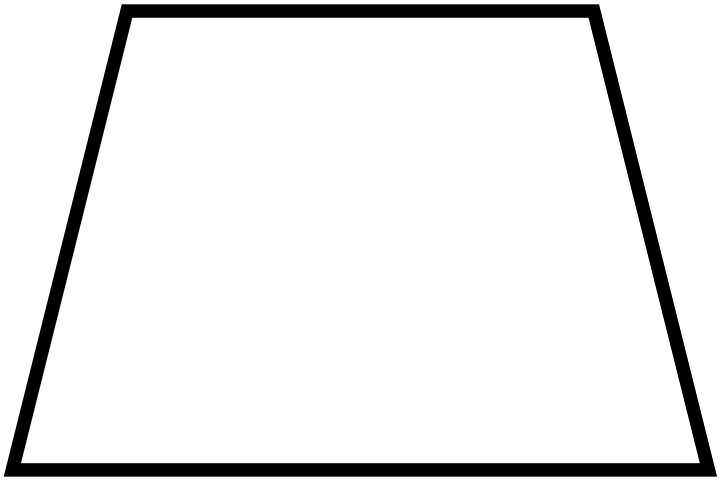
# Triangulate a triangle

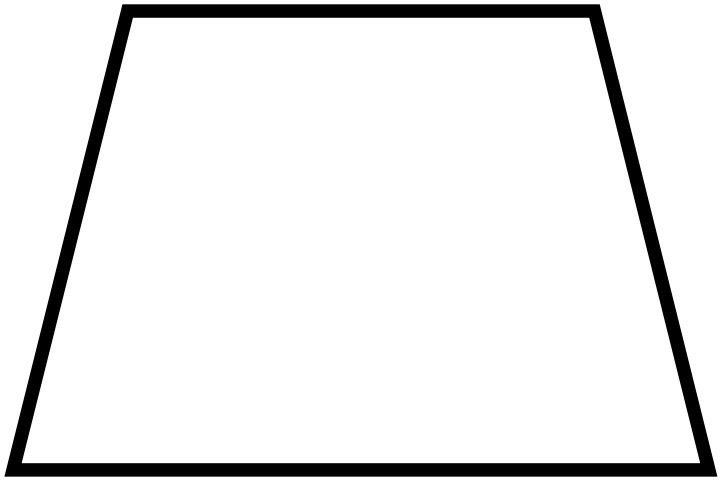
How many ways are there are to triangulate a triangle?



# Triangulate a quadrilateral

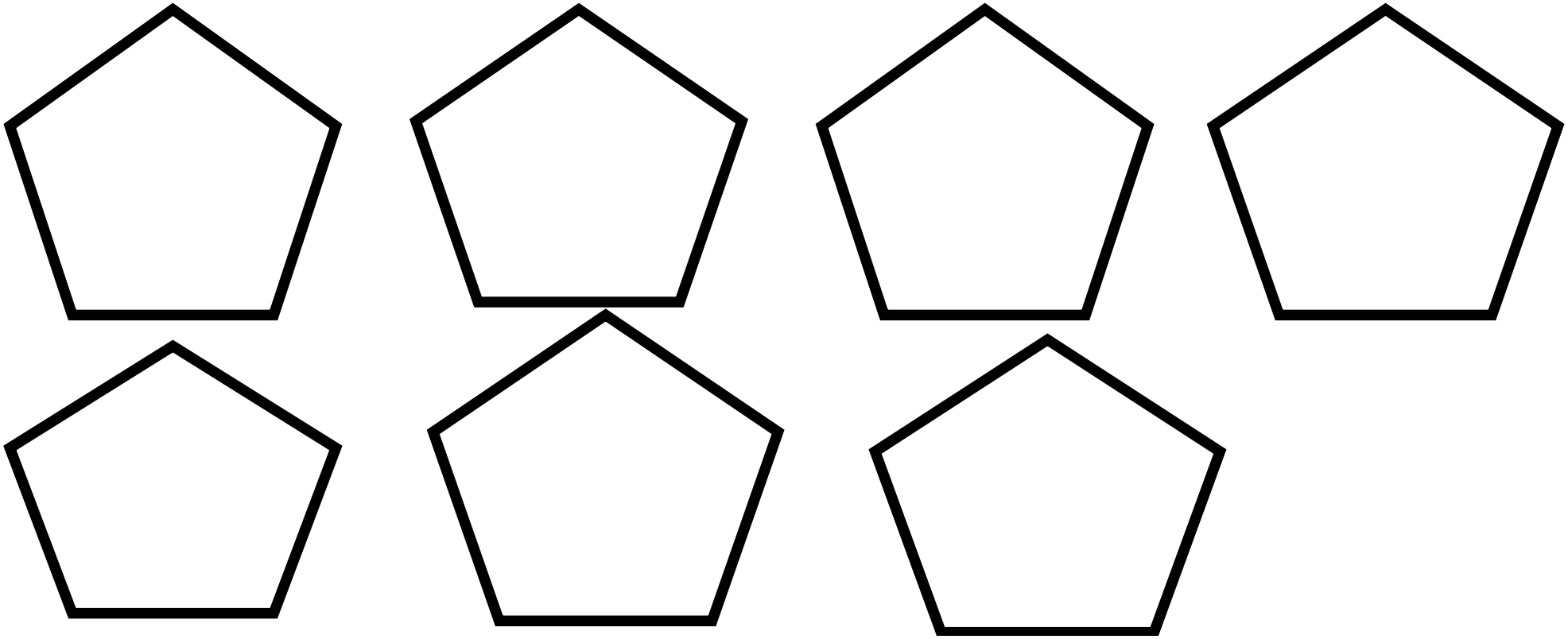
How many ways are there are to triangulate a quadrilateral?





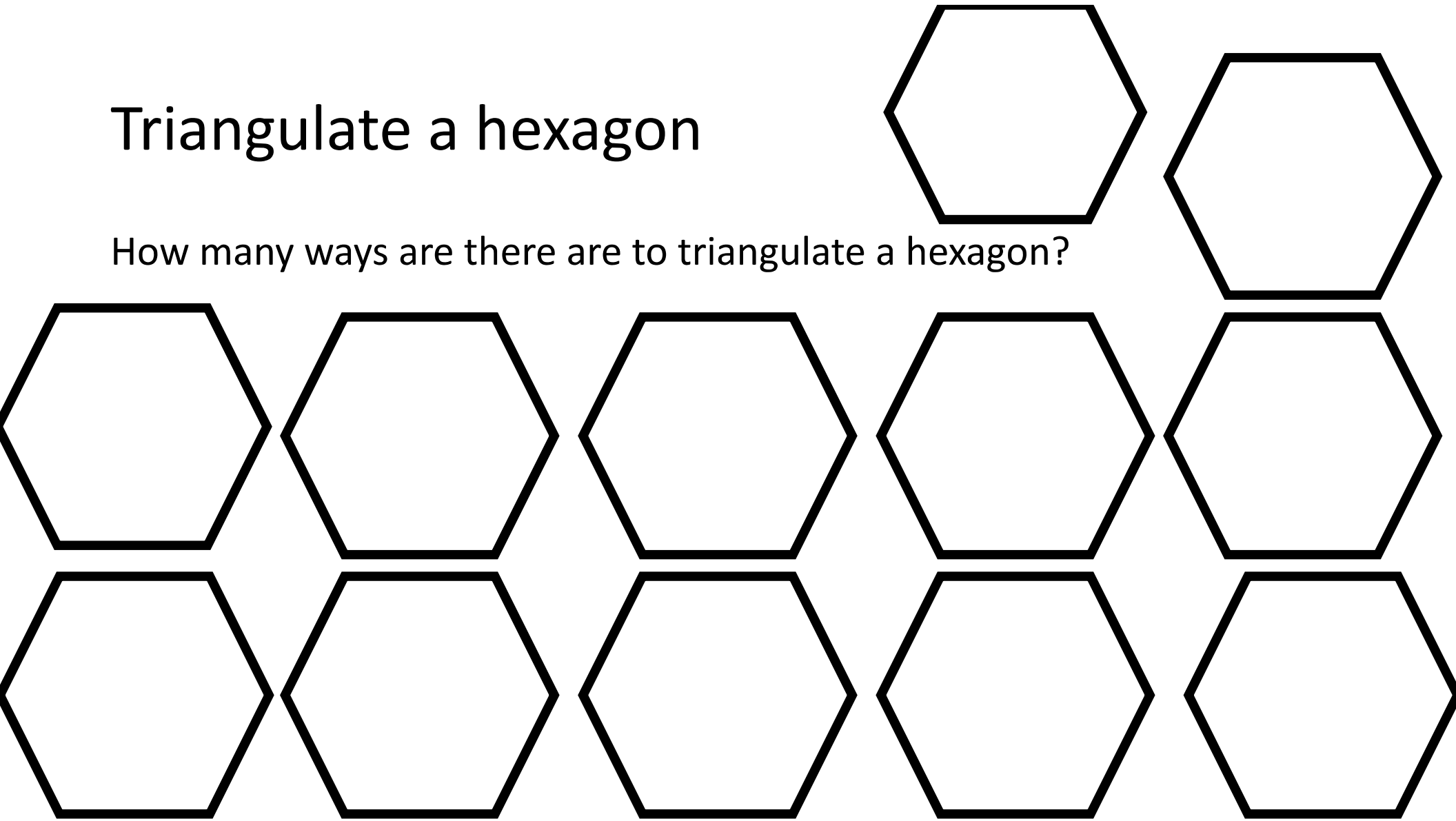
# Triangulate a pentagon

How many ways are there are to triangulate a pentagon?



# Triangulate a hexagon

How many ways are there are to triangulate a hexagon?



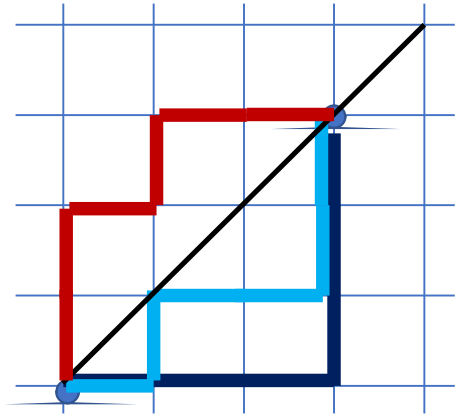
# Breakout Rooms

Now we will look at something completely different (but is it?)

We are going to count how to walk on a grid from (0,0) to (n,n)

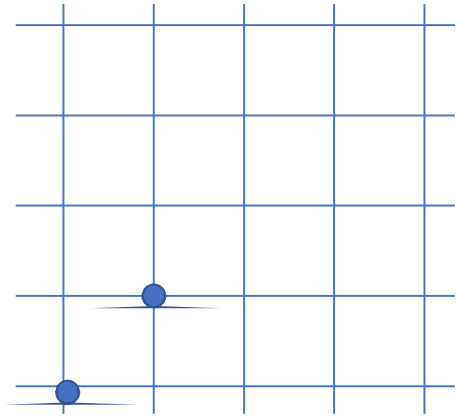
You can go up, and you can go right

However, the y-coordinate can never be greater than the x-coordinate



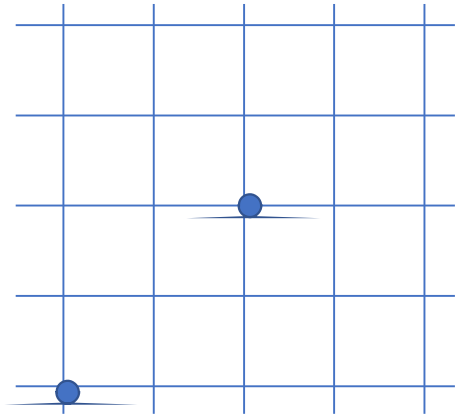
# Count to grid: (1,1)

How many ways are there are to walk to (1,1)?



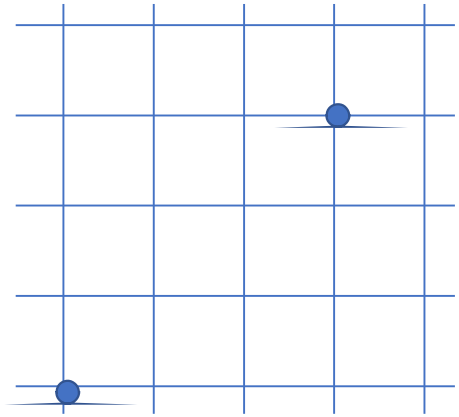
# Count to grid: (2,2)

How many ways are there are to walk to (2,2)?



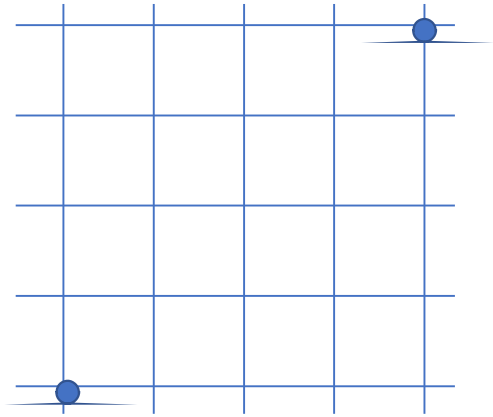
# Count to grid: (3,3)

How many ways are there are to walk to (3,3)?



# Count to grid: (4,4)

How many ways are there are to walk to (4,4)?



# Breakout Rooms

Now we will look at something completely different again (but is it?)

Let's create parenthesis palindromes – we want to find the different ways to pair up parentheses

So – parentheses cannot be unpaired!

This is a valid set: ()()(())

This is not valid: )(()()

# Let’s check in on pairs of parentheses

How many ways are there to create one pair of matched parentheses?

And what about 2 pairs of matched parentheses?

# Let’s check in on pairs of parentheses

How many ways are there to create three pairs of matched parentheses?

And what about 4 pairs of matched parentheses?

# So, what did we learn?

Did you see a pattern in the sequences:

We tried number of triangulations for polygons
polygons were: 3, 4, 5, 6 sided
(remember, their diagonals are: 0,2,3,4)

We tried the grid walks to: (1,1), (2,2), (3,3), (4,4)

And we tried the parentheses sets of : 1, 2, 3, 4

# So, what did we learn?

Those numbers (1,2,5,14) are called Catalan numbers!

$C_0 = 1, C_1 = 1, C_2 = 2, C_3 = 5, C_4 = 14$

Is there an n-th solution?

Yes! But it is more complicated, so we can learn it later.

# Catalan Numbers

The next number for the grid walk was 42.

How many ways do you think there are to triangulate a heptagon?

C_n=(2n choose n)/(n+1)  is the general formula for Catalan Numbers!