

# Math Circle (with a vengeance!)



**Emory Math Circle** 

Have you watched Die Hard? If not, have you watched any spy action movies (like James Bond)?

# The Jug Problem...as told by shaved Bruce Willis

- John McClain (our hero!) and Zeus (Samuel Jackson, not the Greek God) need to disable a very fancy bomb, for which they need exactly 4 gallons of water
- But, they only have a 5-gallon jug and a 3-gallon jug
- There is no way to make any other measurements on the jugs (like the halfway mark), and you cannot estimate
- There is unlimited amount of water in a nearby fountain, and you can pour water between the jugs perfectly without spilling



#### The solution

- Fill only the 5-gallon jug.
- Empty the 5-gallon jug into the 3-gallon jug.
- Now we have 2 gallons left in 5-gallon jug
- Empty the 3-gallon jug.
- Pour the two gallons from 5-gallon jug into the 3-gallon jug.
- Fill the 5-gallon jug again, and pour one gallon from it into the 3-gallon jug
- Now the 3-gallon jug is full
- There are four gallons in the 5-gallon jug!



5-gallon jug



**3-gallon jug** 



#### Your turn

- What about other amounts?
- With the 3- and 5-gallon jugs, can you make 1 gallon? 2 gallons?

#### Your turn

- What about other amounts?
- With the 3- and 5-gallon jugs, can you make 2 gallons?



#### Let's try other jug sizes

- You have an 11-gallon jug and a 4-gallon jug.
- Can you make exactly one gallon?



# Let's try other jug sizes

- You have a 3-gallon and a 6-gallon jug.
- Can you make 4 gallons?



#### Did it work?

- Our hero John McClain always talks things over with his colleagues and friends and peers
- Hint hint

#### Are there quantities you cannot make?

- Why did the previous question not work?
- Can you find the pattern with these additional examples?
  - Make 3 gallons from 2-gallon and 4-gallon jug
  - Make 7 gallons from 10-gallon and 4-gallon jug
  - Make 4 gallons from 3-gallon and 9-gallon jug

#### An egg-celent problem

- You only have a 9-minute egg timer and a 5-minute egg timer (let's forget about why you have \*2\* egg timers)
- How can you boil an egg for 13 minutes?

#### An egg-celent problem

- You only have a 9-minute egg timer and a 5-minute egg timer (let's forget about why you have \*2\* egg timers)
- Can you boil an egg for any arbitrary number of minutes?

# Challenge Problems





#### The art of pouring

- Somehow it matters that we can pour back and forth we have a way of "subtracting."
- What happens when we can't do this?

# Let's get McDonald's

- McDonald's gives McNuggets in packs of 3 and 5
- We are **only** allowed these sizes
- What is the largest number of nuggets we cannot order?





## We visit a few more restaurants

- Wendy's sells nuggets in packs of 3 and 7
- What is the largest number of nuggets we cannot order?



## We visit a few more restaurants

- Burger King sells nuggets in packs of 9 and 14
- What is the largest number of nuggets we cannot order?

#### We visit a few more restaurants

- Waffle House sells waffles in packs of 5 and 10
- What is the largest number of waffles we cannot order?

#### Off to England

- In England, McDonalds sells packs of 6, 9, and 20.
- What's the largest number we can't buy?
- How does this change if they start selling a 4 pack too?









#### These are called Frobenius Numbers!

• The Frobenius number is the largest value *b* for which the following equation as no solution

$$a_1x_1 + a_2x_x + \dots + a_nx_n = b$$

- Here, a<sub>i</sub> are positive integers, b is an integer, and x<sub>i</sub> are non-negative integer solutions
- You can read more about them here:

Frobenius Number -- from Wolfram MathWorld

# Yippie-Kai Yay, Mr. Falcon

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