Some fun Graphs!!

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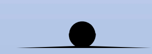
If you could wear a facemask for the rest of your day today, what colour would you want it to be?

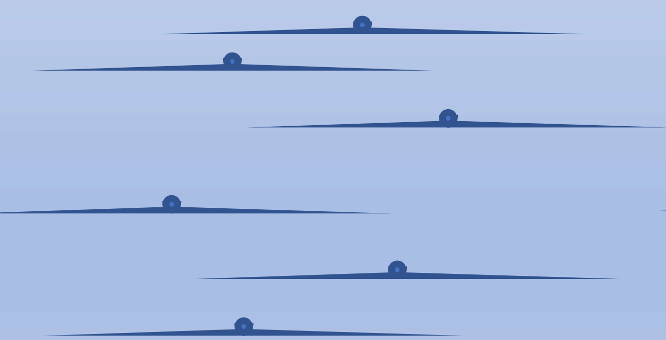
The most basic graph

- The basic graph is a single vertex
- What is V, E, and F here?
- V = number of vertices
- E = number of edges
- F = number of faces

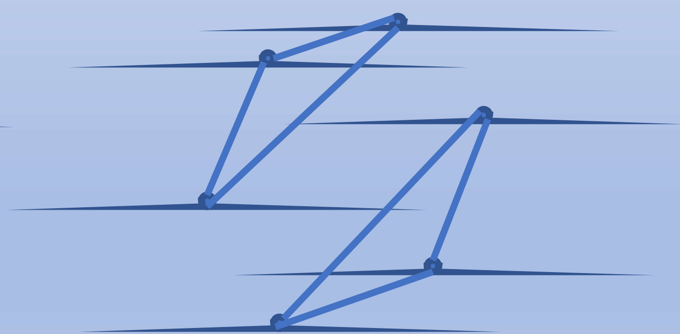


Vertices, Edges, and Faces

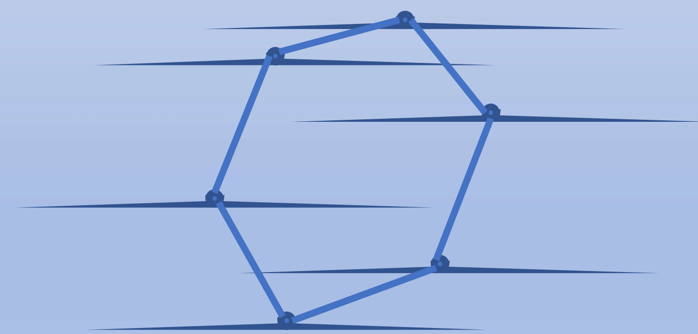
- Left Graph: 6 vertices, 0 edges, 1 face
- Middle Graph: 6 vertices, 6 edges, 3 faces
- Right Graph: 6 vertices, 6 edges, 2 faces



Vertices



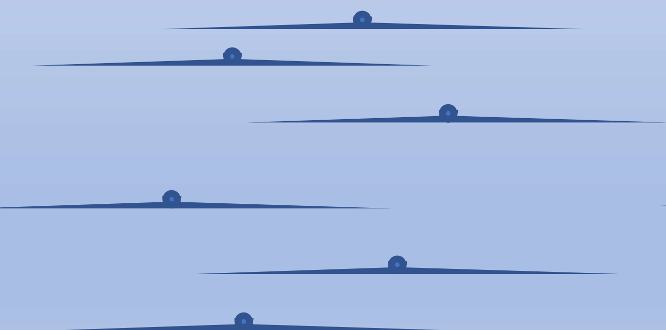
Graph



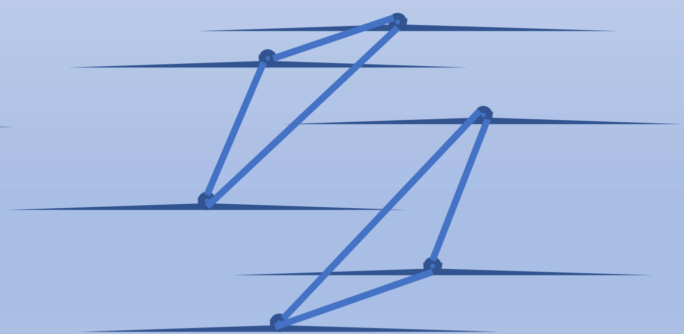
Connected Graph

Graphs and Connected Graphs

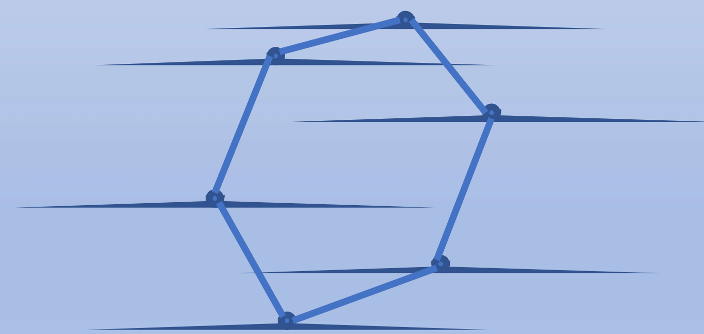
- A graph is when we have some vertices, and connect them with edges
- A connected graph means you can go from any point to any other point by traveling through edges



Vertices



Graph



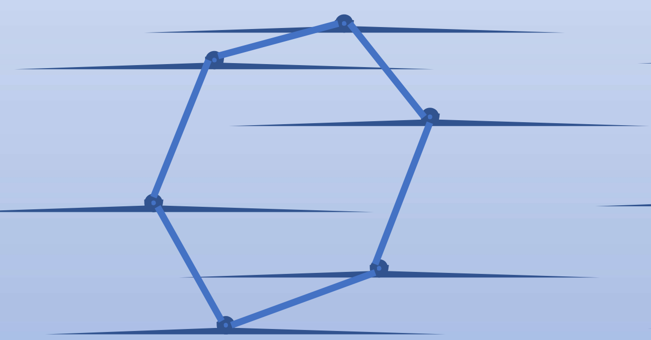
Connected Graph

Making your own

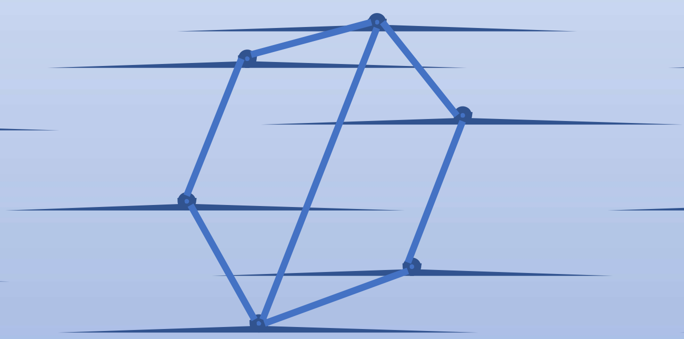
- Draw a connected graph with only 1 edge
- Calculate V, E, and F

Planar Graphs and Faces

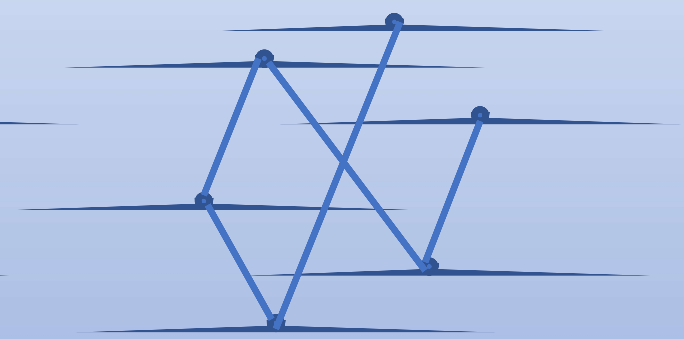
- A graph where no edges cross each other is a planar graph



Planar Graph



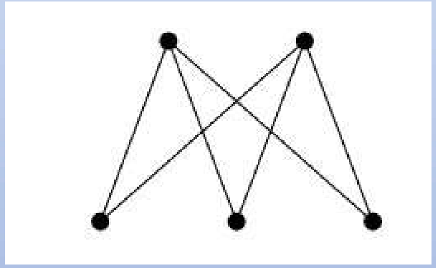
Planar Graph

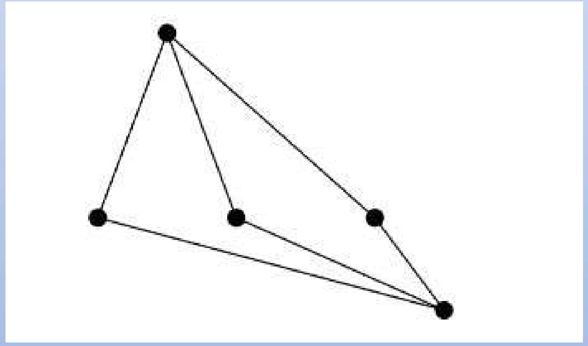


Non-Planar Graph

Redrawing Planar Graphs

- You can sometimes rearrange how edges are drawn to make a graph planar

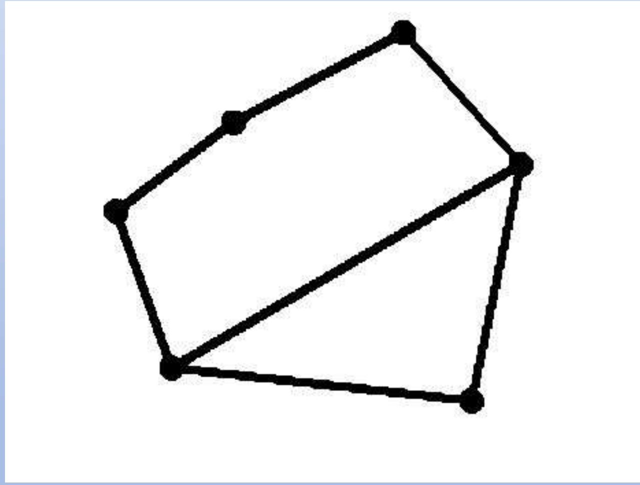




Breakout Rooms

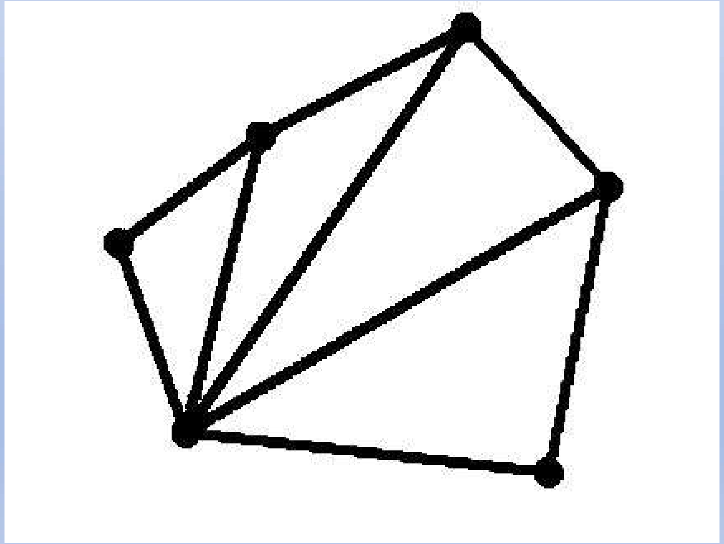
Let’s Explore Planar Graphs by Counting

- How many vertices are here?
- How many faces?
- How many edges?

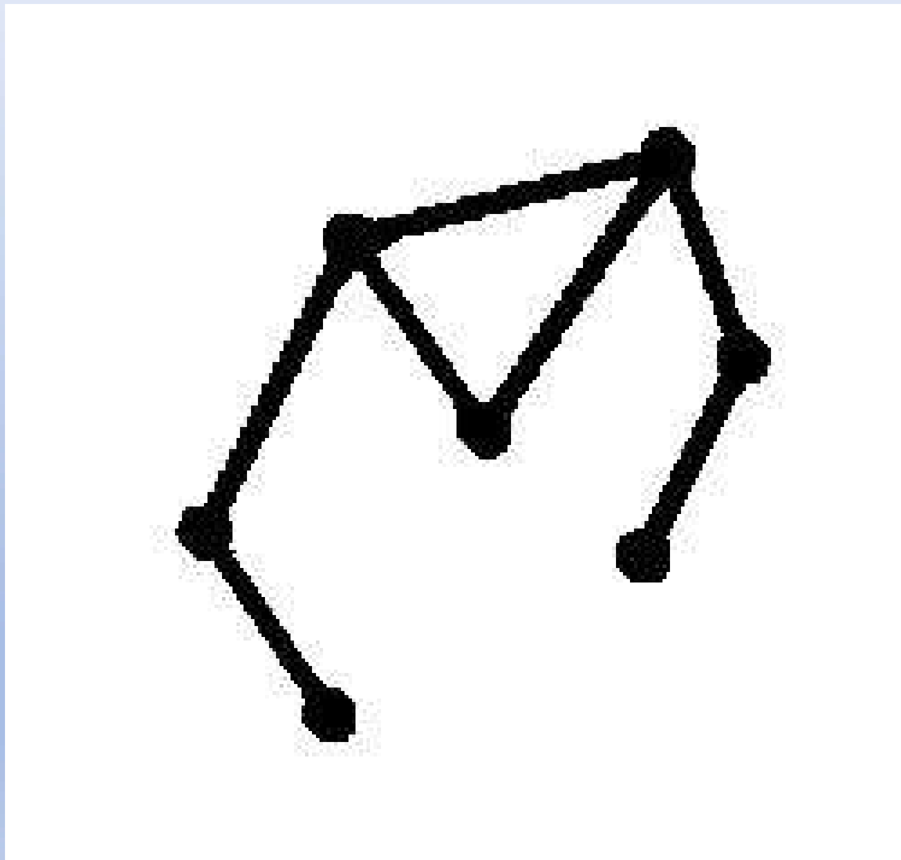


Let’s add some edges

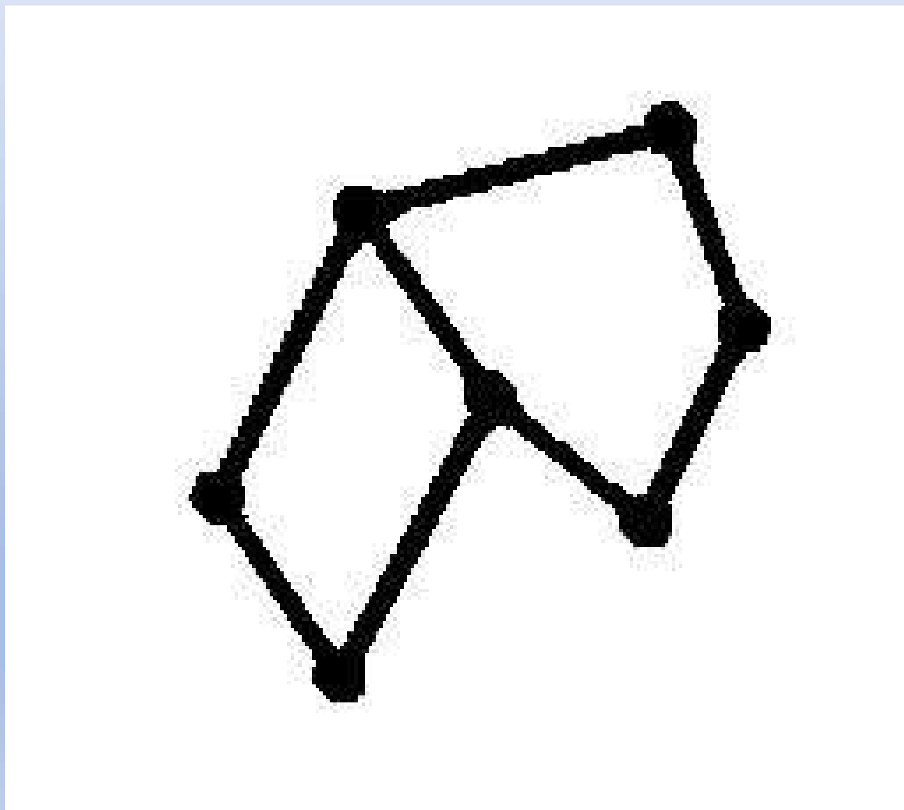
- How many vertices are here?
- How many faces?
- How many edges?



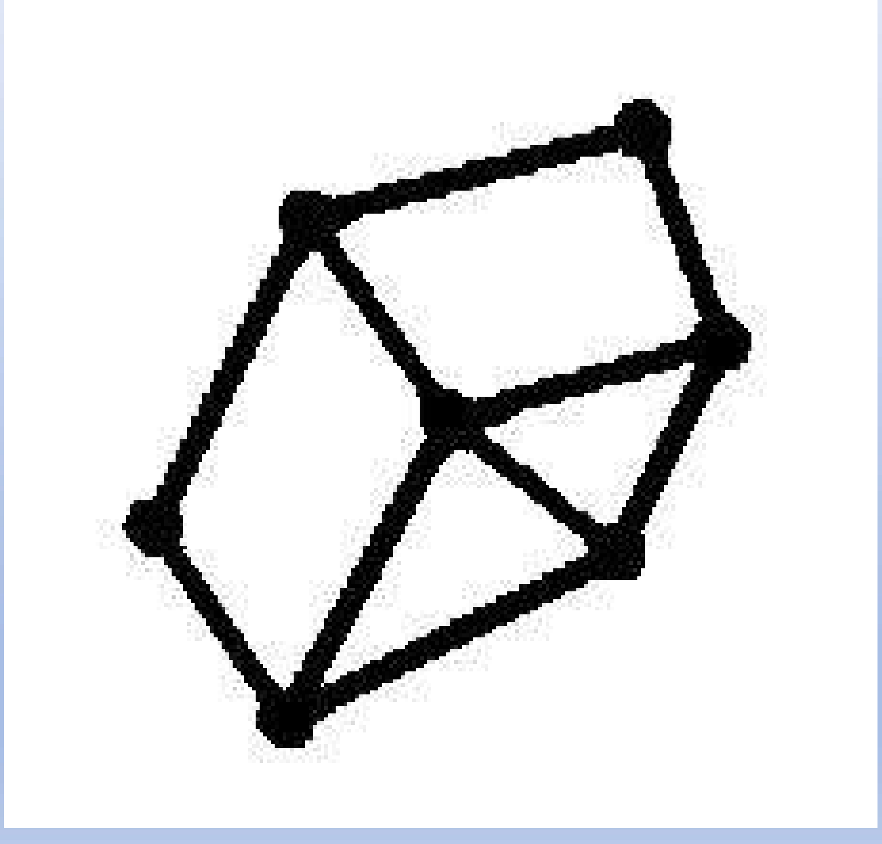
Let’s try a few… Graph1



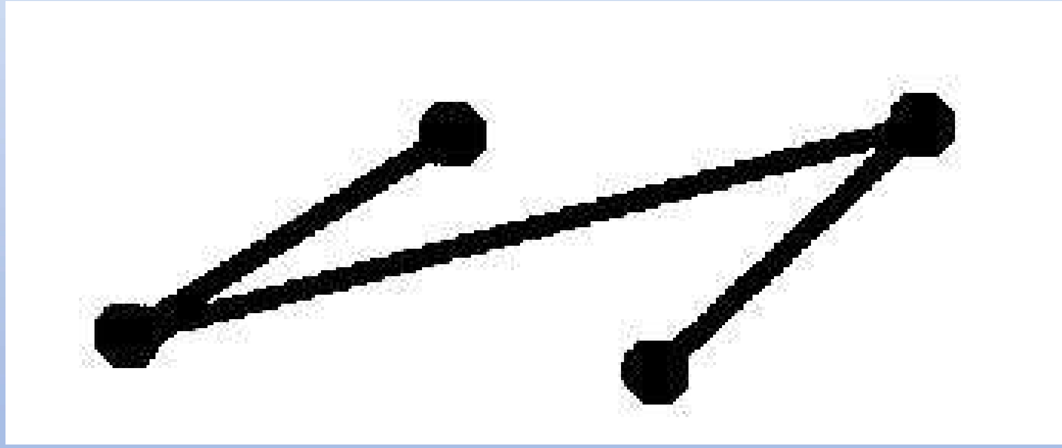
Graph2



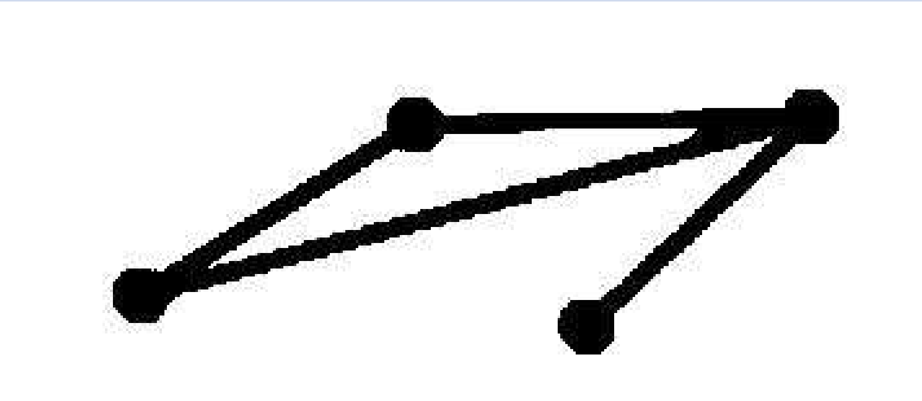
Graph3



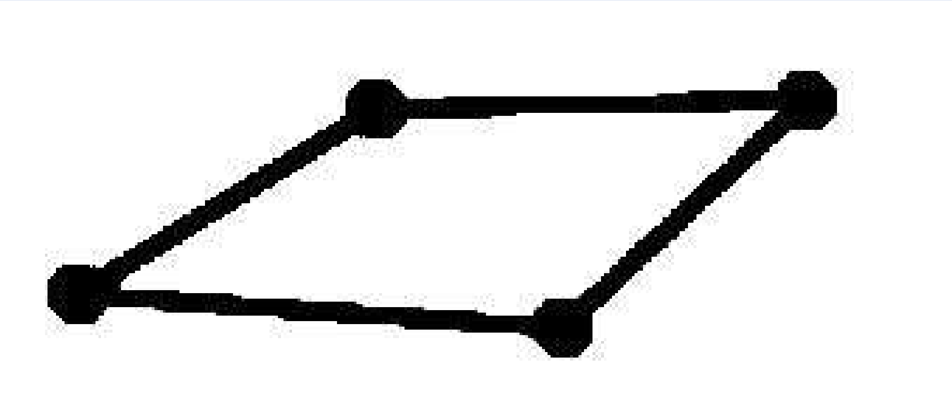
And a few more.. Graph4



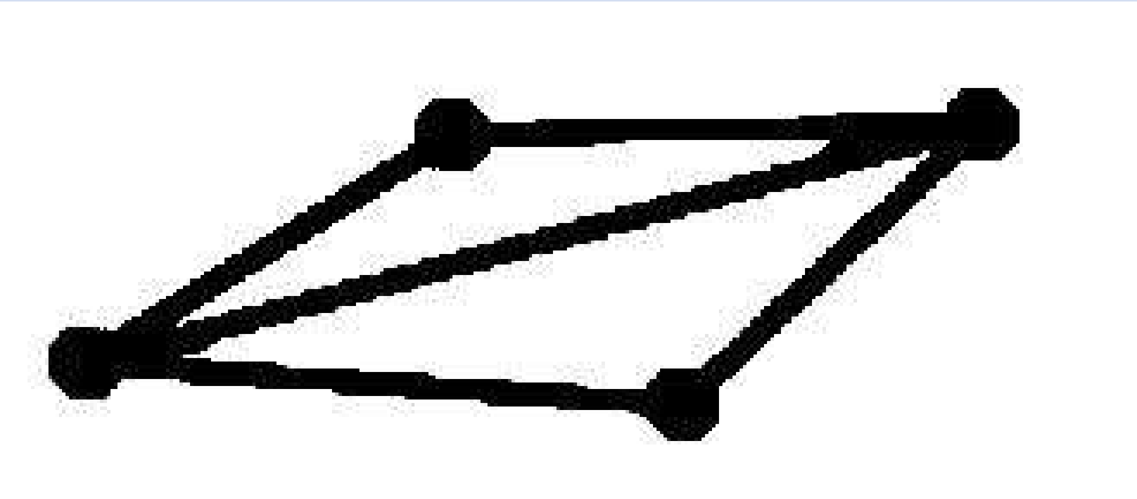
Graph5



Graph6



Graph7



Time to reflect..

- Go back to all the previous graphs you worked on and answer this:
  Are there usually more vertices, edges, or faces?

Time to reflect..

- Go back to all the previous graphs you worked on and answer this:
  Were there more edges than the faces and vertices combined?

Is there a relationship between the number of edges, faces and vertices?

Euler Characteristic

Definition: It is defined as V – E + F for any connected planar graph.

Making your own II

- Draw a connected graph with only 2 edges
- Calculate V, E, and F
- What is the Euler characteristic?

Three-Edge graphs

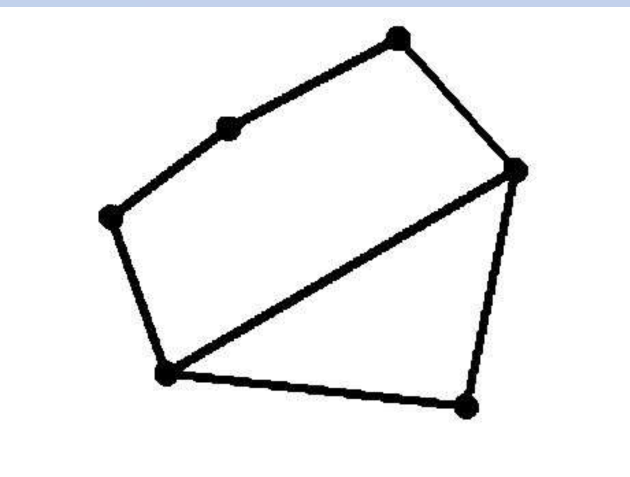
- Can you draw three connected graphs, each with three edges?
- Can you make any more?
- What is the Euler characteristics of each $(V - E + F)$

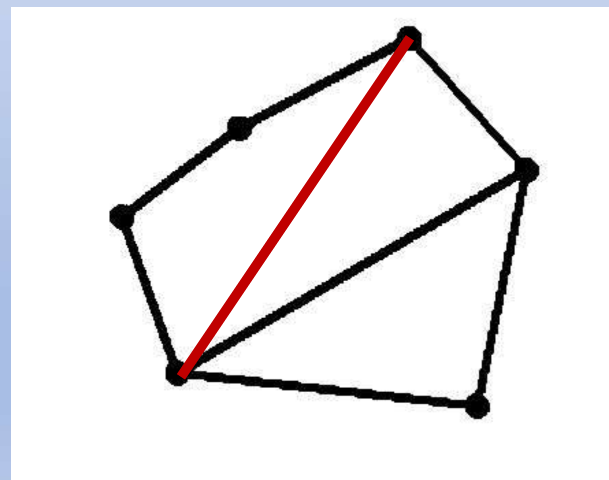
Adding a vertex and edge

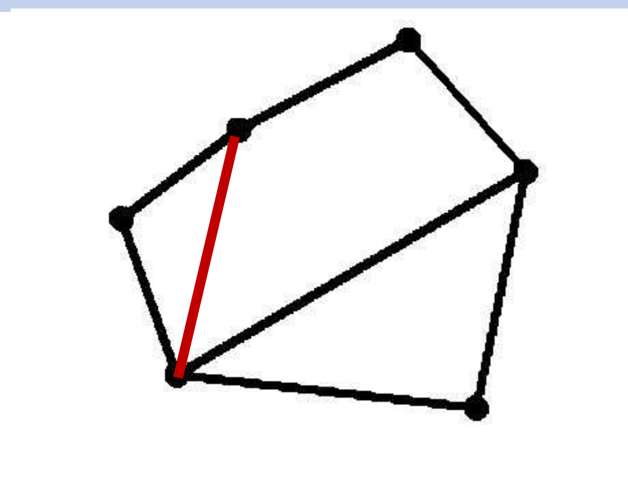
- Let's take any of your graphs you have made
- Add a vertex somewhere outside, then connect it to another vertex without crossing any edges
- What is the new Euler characteristic $(V - E + F)$?

Adding a new edge

- Let's take the graph below. We'll add a new edge between 2 vertices.
- How does this change the Euler characteristic ($V - E + F$) in each case?







Time to conjecture…

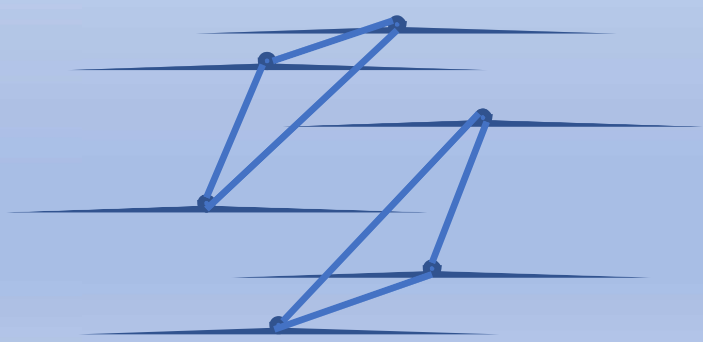
- Do you notice a pattern? Can you conjecture something about the Euler characteristic?

Non-intersecting 4 points

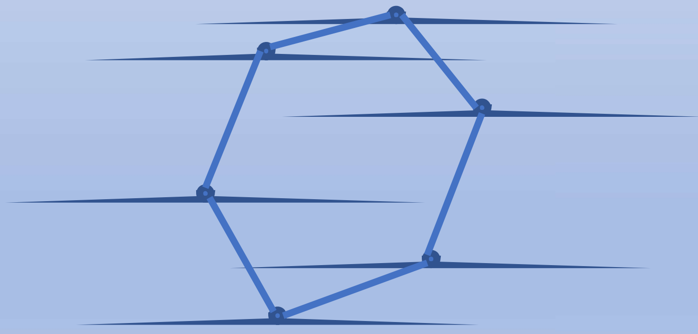
- Can you draw 4 vertices so that when you connect every pair of points, none of them cross each other?

Unconnected graphs

- So far, we have worked with connected graphs.
- What are the Euler characteristics ($V - E + F$) of the following unconnected graph?
- Compare it to the connected graph's Euler characteristics.



Unconnected Graph



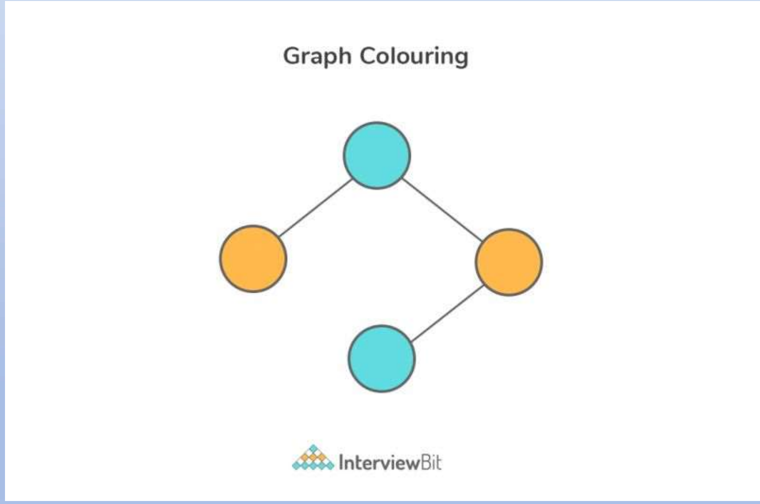
Connected Graph

Moving to the real world

- Say there are 10 cities, and 16 highways connecting them
- None of the highways cross each other.
- Let’s say the space between highways and cities are farms
- How many farms are there?
- If we follow the rules above, will the number of farms ever change?

The Graph Colouring problem…

- The process of assigning colors to the vertices such that no two adjacent vertices have the same color is caller Graph Colouring.



The Graph Colouring problem…

- **Chromatic Number:** The smallest number of colours needed to colour a graph G is called its chromatic number.
- For example, in the above image, vertices can be coloured using a minimum of 2 colours.
- Hence the **chromatic number** of the graph is 2.

Four colour theorem

- A very famous result about graphs and colouring the vertices of graphs is called the *Four Colour Theorem*!
- Look it up if you want to know more!

THANK YOU!!