

Welcome to Math Circle

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Middle School C

Rules for Math Circle

- So...what do *you* think the rules should be?
- Type them in the chat!

Warmup

- Ariel, Bill, Carla (A,B, and C) are standing in a line. How many different ways can they do this?



Warmup

- Let's say they are standing in the order A, B, C
- The person in position 1 swaps with the person in position 2. What does the line look like now?

Warmup

- If we keep swapping, how many times do we need to repeat this get back to the order A, B, C?

Breakout Room

Scenario 1

- Everyone goes back to the starting position A, B, C.
- This time, everyone moves one position to the right (so 1 moves to 2, 2 moves to 3, and 3 moves to 1).

Scenario 1

- Draw this and repeat the process several times. What happens?

Scenario 2

- Now Darryl (we call him D) joins in.



Scenario 2

- How many different configurations of A, B, C, D are there?

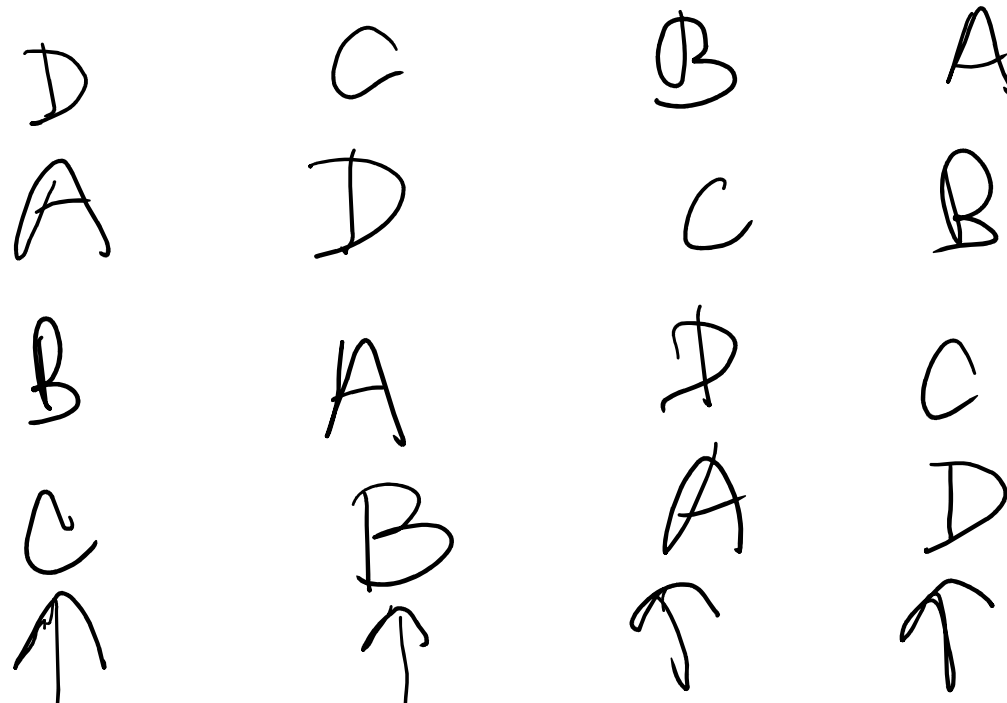
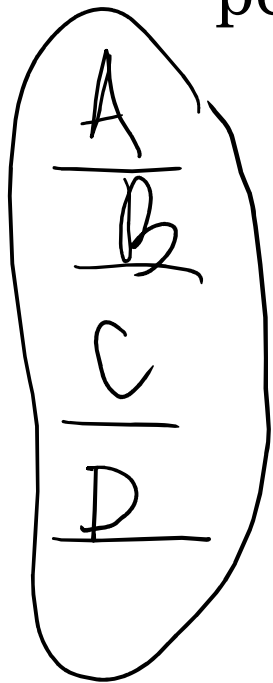
A B D C

A C D B

1 → 2 2 → 3 3 → 4 4 → 1

Scenario 3

- Try warmup (swapping) again, with the four people. Is anything different? Why or why not?

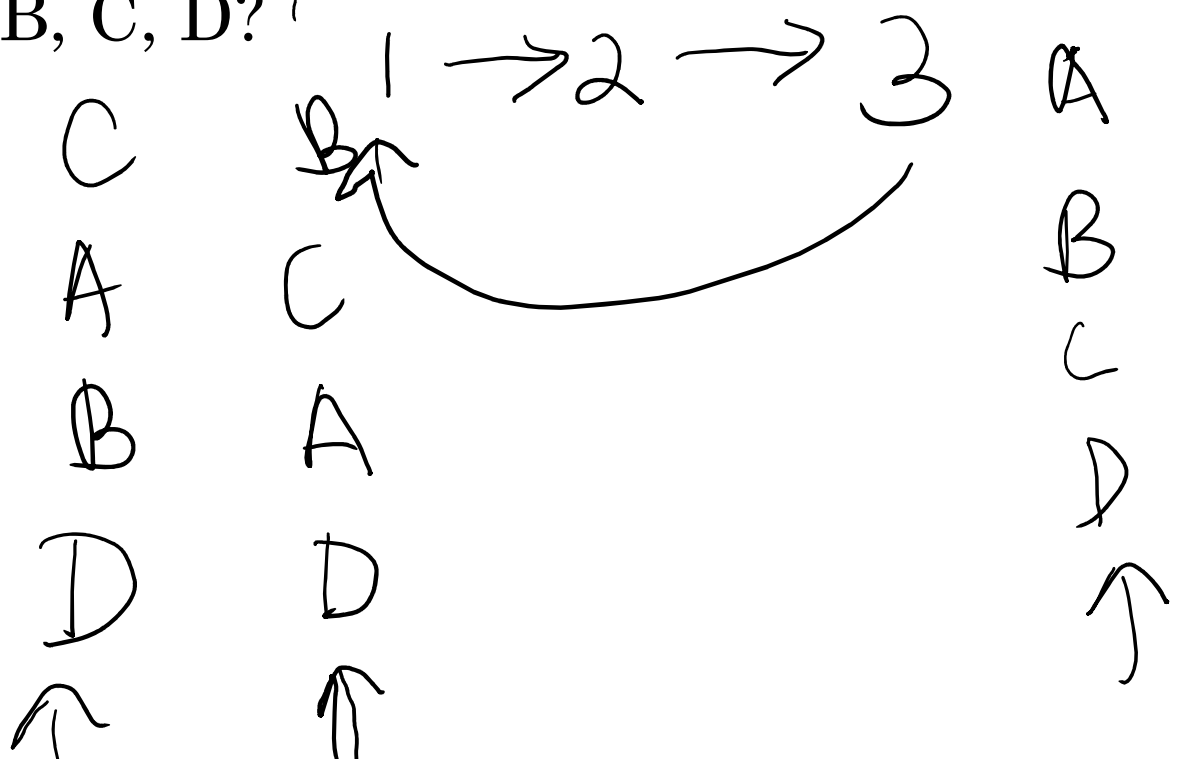
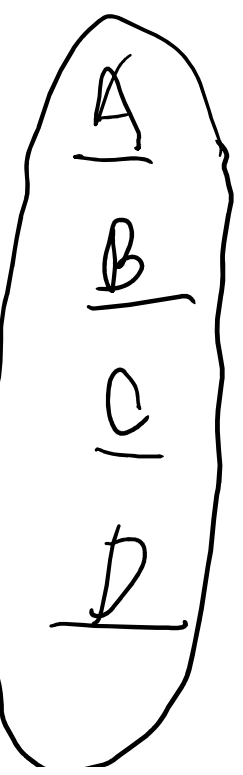


① 1 ↔ 4 ② 1 ↔ 3 ③ 2 ↔ 3

Scenario 4

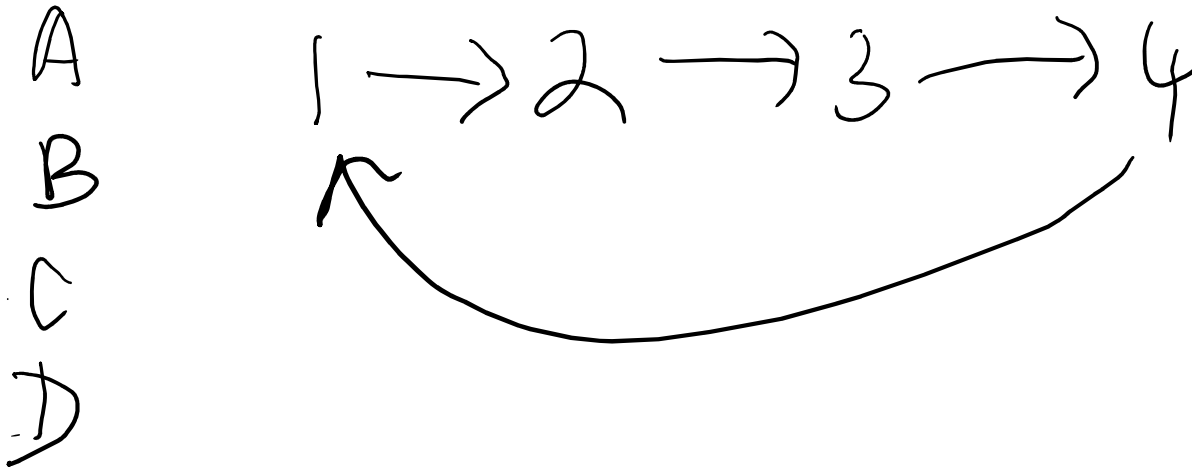
④ 3 ↔ 4

- Can you come up with another rearrangement of A, B, C, D?



Scenario 4

- ✓ If you repeat it several times, how long does it take to get back to the start?
- Can you find a rearrangement that takes the longest?



Challenges!!! The First

- What if more people join in? Try some of these with 5, 6, or more people:
- ✓ • How many different ways can they stand in a line?
- ✓ • Can you find a rearrangement that takes the longest to repeat?

Challenges!!! The Second

- Draw a triangle with equal sides and number its points 1, 2, 3. How many ways can you rotate or flip this triangle? Compare this with rearranging 3 people standing in a line.

Challenges!!! The Second

- Draw a square and label its four points 1, 2, 3, 4. How many ways can you rotate or flip it? Compare this with rearranging four people standing in a line.

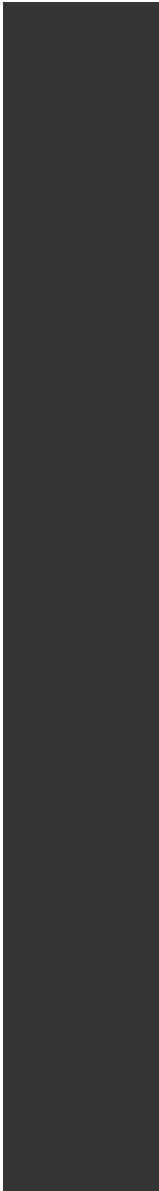
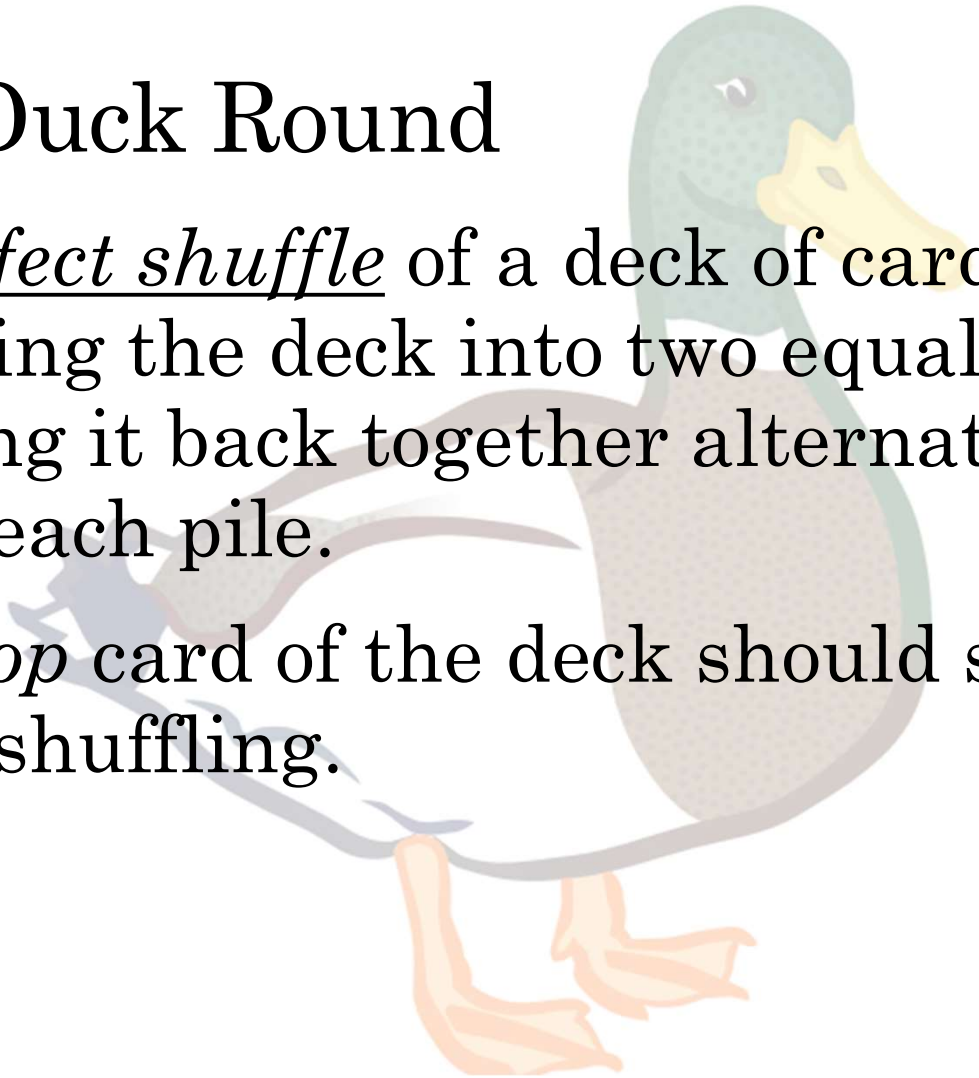
Challenges!!! The Third

- Can you come up with a rule for the following:
- How many ways n people can stand in a line (here n is a number)?

- How many ways we can rotate or flip an n -sided polygon with equal sides?

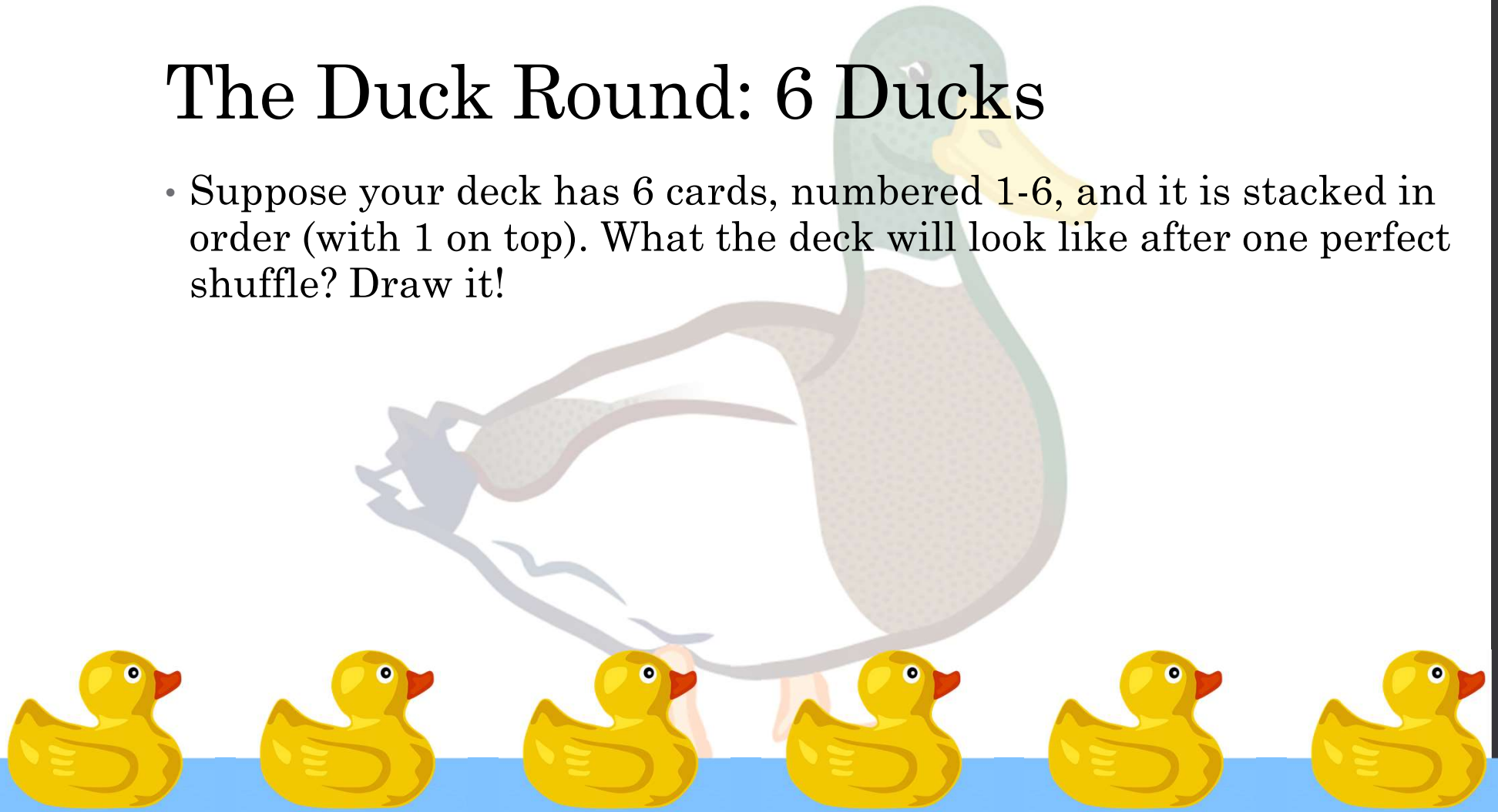
The Duck Round

- A perfect shuffle of a deck of cards consists of splitting the deck into two equal parts and putting it back together alternating one card from each pile.
- The *top* card of the deck should stay on top after shuffling.



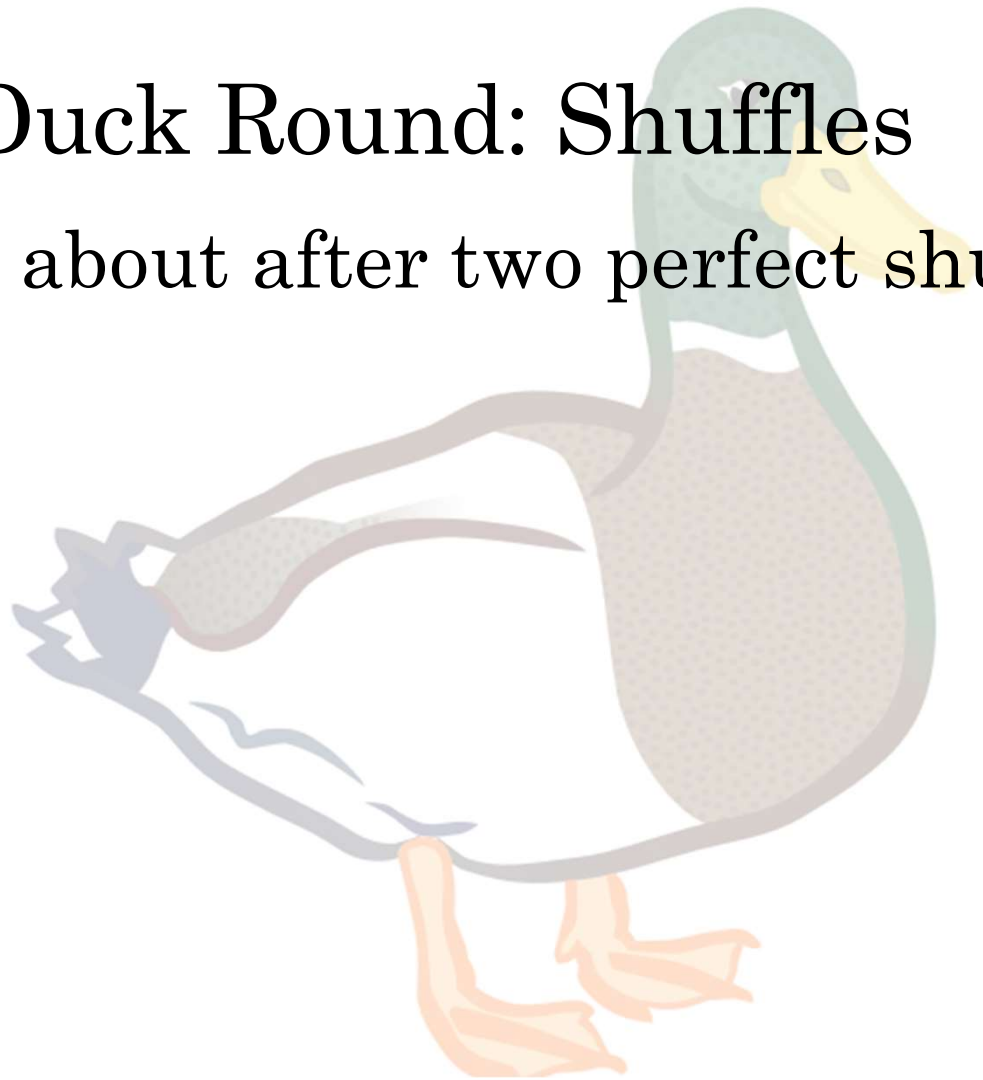
The Duck Round: 6 Ducks

- Suppose your deck has 6 cards, numbered 1-6, and it is stacked in order (with 1 on top). What the deck will look like after one perfect shuffle? Draw it!



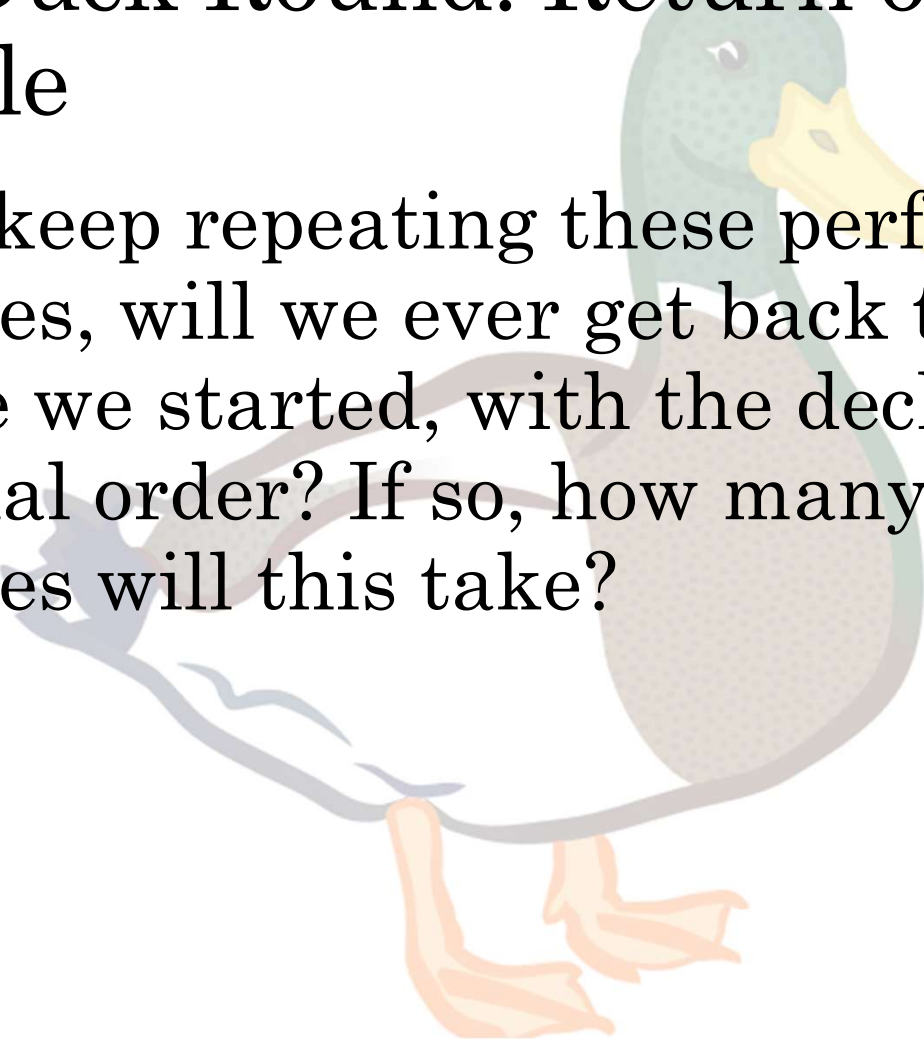
The Duck Round: Shuffles

- What about after two perfect shuffles?



The Duck Round: Return of the Shuffle

- If we keep repeating these perfect shuffles, will we ever get back to where we started, with the deck in its original order? If so, how many shuffles will this take?



The Duck Round: More Ducks

- Repeat the previous questions with larger decks (you might try 8, 10, and 12 cards). Do you notice any patterns?

